

# Legacy vs. SIGMET dBZ Calculation

In this paper, we discuss how the weather radar equation is interpreted between legacy and SIGMET. This difference shows up in the different ranges for SYSCAL (legacy) and dBZ<sub>0</sub> (SIGMET), but model the exact same equation. This paper will show how each is derived, and give nominal values for each.

The legacy values shown are well validated; however, we still have to validate the SIGMET numbers shown on the WSR-88D through calculations of dBZ<sub>0</sub>.

## Radar Equation:

The basic weather radar equation makes the assumption of Gaussian returns with Rayleigh scattering (a large number of small points in the radiated volume).

The general weather radar equation for Z<sub>en</sub> is (using standard metric units and organized in a “NEXRAD friendly” format):

$$Z_e = P_R \times R^2 \times \frac{1}{L_P} \times \frac{2^{10} \times \ln(2) \times \lambda^2}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L}$$

**Figure 1, Radar Equation**

Z is normally cast as  $\frac{mm^6}{m^3}$  so Z<sub>en</sub> needs to be converted from m<sup>3</sup> to those units (a factor of 10<sup>18</sup>).

The main difference between legacy and SIGMET is how they handle the P<sub>R</sub>, the receive power. In the radar equation, P<sub>R</sub> is Signal only. However, in the radar Return the signal processor sees, noise is mixed with the Signal and must be accounted for to get an accurate Z<sub>en</sub>. Also, since we are using A/D converters on the signal voltage (not power) and our receiver changes the gain of P<sub>R</sub>, these must also be accounted for (a<sup>2</sup>g accounts for this in legacy)

The legacy system uses P<sub>R</sub> as (Return - Noise), while Signet’s model for P<sub>R</sub> is (Return – Noise)/Noise, the signal to noise ratio. This difference is accounted for in the calibration constants used by legacy (SYSCAL) and SIGMET (dBZ<sub>0</sub>).

The effect of this is that Signet’s calibration constant, dBZ<sub>0</sub>, differs from SYSCAL by the Noise value (around 50dB in the WSR-88D) and the difference in receiver gain.

Table 1 shows the parameters associated with this paper, and the common units these parameters are displayed in. The radar equation used assumes standard metric units, so to use these parameters they must be converted to standard units first (i.e. cm converted to m in the radar equation).

**Table 1, Parameters**

Symbol	Name	Units
λ	Wavelength	cm
π	Pi	Unitless
K	Refractivity	Unitless
P <sub>R</sub>	Receive Power	mW

P <sub>T</sub>	Transmit Power	kW
N	Noise	mW
R	Range	km
L	Losses (except propagation loss, see table 2)	Unitless
τ	Pulsewidth	μsec
G	Antenna Gain	Unitless
c	Speed of Light	m/sec
K	Refractivity	Unitless
ln	Natural Logarithm	Unitless
L <sub>p</sub>	2 way atmospheric propagation loss	Unitless
g	Receiver Gain	Unitless
θ	Beamwidth	Radians

**Table 2, Losses**

Symbol	Name	Description	Nominal Value (dB)
L <sub>t</sub>	Transmitter	Transmitter Waveguide Loss	2.5
L <sub>r</sub>	Receiver	Receiver Waveguide Loss	.63
L <sub>d</sub>	Detection	Receiver Detection Loss	1.5
g	Receiver Gain	Receiver gain from Receiver Protector to A/D conversion	Legacy: 55dB ORDA: 39dB

## Legacy Casting:

$$dBZ = 10 \log(P_R - N) + 20 \log(R) - 10 \log(L_p) + SYSCAL$$

$$SYSCAL = 10 \log \left( \frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^6 \times 10^{-3}}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{1}{a^2 g} \right)$$

## SIGMET Casting:

$$dBZ = 10 \log \left( \frac{P_R - N}{N} \right) + 20 \log(R) - 10 \log(L_p) + dBZ_0$$

To calculate dBZ<sub>0</sub>, we calculate the radar equation for signal power equal to the noise power at 1km. We'll put this into Figure 1, Radar Equation with the following definitions:

$$P_R = \left( \frac{2N - N}{N} \right) = 1 \quad (\text{signal power equal to noise power})$$

Since P<sub>R</sub> has Noise in the denominator, it must be in the numerator somewhere in the radar equation as well R=1km (since we cast R in terms of km, this will cancel out the range)

L<sub>p</sub>=1 (since we're using a test signal to calculate dBZ<sub>0</sub>, there's no correction for atmospheric gas)

Unit changes: Z conversion to  $\frac{mm^6}{m^3}$  is 10<sup>18</sup>, P<sub>R</sub> conversion to mW is 10<sup>-3</sup> and m<sup>2</sup> conversion to km<sup>2</sup> is 10<sup>6</sup>.

Z<sub>0</sub> becomes:

$$Z_0 = \left( \frac{2N - N}{N} \right) \times 1^2 \times \frac{1}{1} \times \frac{2^{10} \times \ln(2) \times \lambda^2}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L} \times (10^{18} \times 10^{-3} \times 10^6) \times N$$

Since L is equal to the product of the losses, and our reference point for measurements is the input to the Receiver Protector, L is split up as follows:

$$L = (L_t \times L_r \times L_d) \times g$$

Now we convert Z to dB:

$$dBZ_0 = 10 \log(P_R) + 20 \log(R_0) - 10 \log(L_P) + 10 \log \left( \frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^{-3} \times 10^6}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{N}{g} \right)$$

$$dBZ_0 = 10 \log \left( \frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^{-3} \times 10^6}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{N}{g} \right)$$

## SYSCAL – dBZ<sub>0</sub> Comparison

The difference between SIGMET and legacy in the radar equation is how P<sub>R</sub> is handled. In the constants, the  $\frac{N}{a^2 g}$  term is the difference between legacy and SIGMET calibration constants. The g term (receiver gain from front end to A/D conversion) is used here because P<sub>R</sub> is referenced to this point (front end insertion).

## SYSCAL interpretation:

These terms are related to how P<sub>R</sub> is represented in the radar equation. For legacy, the P<sub>R</sub> term is in digits<sup>2</sup> from the A/D converter without being converted to voltage.

The g term is the same for both, representing the receiver gain. In legacy, the a<sup>2</sup> term represents conversion from A/D digits to power (voltage<sup>2</sup> represents power, and since a represents volts/digit, a<sup>2</sup> represents power/digit<sup>2</sup>).

## dBZ<sub>0</sub> interpretation:

dBZ<sub>0</sub> is used as a signal reference that all reflectivity is compared against. It represents the signal 3dB above the noise (Signal=Noise) at a range of 1km.

For SIGMET, I and Q values are converted to voltage before pulse pair and power calculations. Therefore dBZ<sub>0</sub> does not have to account for the a<sup>2</sup> term.

SIGMET models P<sub>R</sub> as the Signal to Noise ratio, therefore putting N in the calibration constant.

## Calculations of SYSCAL and dBZ<sub>0</sub>

Table 3 shows the calculations of SYSCAL and dBZ<sub>0</sub> using typical values for various components. Numerator values are above the heavy line (at Noise value), and denominator values are below. In dB, this changes the sign. The column “Radar Equation Value” shows the number needed for the radar equation. This takes into account where we need exponentiation of a value (for example, the radar equation uses the wavelength ( $\lambda$ ) squared, and the Radar Equation Value column is the square of the value column). Refer to the Radar Equation above to determine the exponentiation used.

**Table 3, SYSCAL and dBZ<sub>0</sub>**

Term	Units	Value	Radar Equation Value	dB	SYSCAL	dBZ <sub>0</sub>
2 <sup>10</sup>	Unitless	1024.00	1024.00	30.10	30.10	30.10
ln(2)	Unitless	0.693147	0.693147	-1.59	-1.59	-1.59
wavelength	cm	10	100	20.00	20.00	20.00
cm to m conversion	m	0.01	0.0001	-40.00	-40.00	-40.00
m <sup>3</sup> to mm <sup>6</sup> /m <sup>3</sup> conversion		1.00E+18	1.00E+18	180.00	180.00	180.00
milliwatts to W conversion	W	0.001	0.001	-30.00	-30.00	-30.00
km to m conversion	m	1000	1000000	60.00	60.00	60.00
Noise	mW			-77		-77
antenna gain	Unitless			-91.60	-91.60	-91.60
antenna beamwidth	degrees	0.91	0.8281	0.82	0.82	0.82
degrees to radians conversion	radians	0.017453	0.000305	35.16	35.16	35.16
Pulsewidth	Microseconds	1.57	1.57	-1.96	-1.96	-1.96
microseconds to seconds conversion	sec	0.000001	0.000001	60.00	60.00	60.00
speed of light	m/sec	3.00E+08	3.00E+08	-84.77	-84.77	-84.77
refractivity	Unitless	0.93	8.65E-01	0.63	0.63	0.63
pi	Unitless	3.14159	31.0062	-14.91	-14.91	-14.91
transmitted power	kW	700	700	-28.45	-28.45	-28.45
kW to W conversion	W	1000	1000	-30.00	-30.00	-30.00
transmitter waveguide loss	Unitless			2.25	2.25	2.25
receiver waveguide loss	Unitless			0.63	0.63	0.63
receiver detection loss	Unitless			1.5	1.5	1.5
a2g (legacy)	Unitless			-57.00	-57.00	
receiver gain (SIGMET)	Unitless			-38.75		-38.75
Total					10.81	-47.94

The SYSCAL value 10.76 agrees well with the field measured SYSCAL of 10.627 (average SYSCAL from all field systems before installation of EMI Filter, standard deviation is 1.10).

The SIGMET receiver gain is the gain from the Receiver Protector 2A3 to the input of the SIGMET IFD, including the 2db insertion loss from Signet’s anti-aliasing filter. This differs from legacy significantly because of removal of the IF and analog portions of the legacy receiver (the gain is just to the output of the Mixer/Preamplifier 4A5)

## **Conclusion:**

Both legacy and SIGMET use the same equation for reflectivity, they just model it differently. All the terms are accounted for, and therefore there will be no difference in reflectivity between legacy and SIGMET.